

# **Basis Generator Method Calculations for Ion-Atom Collisions of Indirect and Direct Relevance to Neutral Beams in Fusion Plasmas**

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# Two (not so) easy pieces

Lyman line ratios in  
charge-exchange  
collisions of C<sup>6+</sup> and  
O<sup>8+</sup> ions with **hydrogen**  
**and krypton** atoms

*Phys. Rev. A* **97**,  
062705 (2018)

Target excitation,  
electron capture, and  
ionization in **p-H( $n = 2$ )**  
collisions

(work in progress)

(A)

(B)

# Outline

1. **Intro ✓**
2. **Theory** : brief recap of treatment of few-electron problem and two-center basis generator method (TC-BGM)
3. **Study #1**: collisions with hydrogen vs. krypton atoms
4. **Study #2**: collisions with excited hydrogen atoms
5. **Summary and outlook**

## **2. Theory: the few-electron problem and the TC-BGM**

# Few-electron problem I

- Explicit solution is hard
- Ansatz (independent electrons – IEM):

$$\hat{H}_e(t) \rightarrow \sum_{j=1}^N \hat{h}_j(t), \quad i\partial_t \psi_j(\mathbf{r}, t) = \hat{h}(t) \psi_j(\mathbf{r}, t)$$

$$\hat{h}(t) = -\frac{1}{2}\Delta - \frac{Z_T}{r} - \frac{Z_p}{|\mathbf{r} - \mathbf{R}(t)|} + v_{ee}(\mathbf{r}, t)$$

- Choice of  $v_{ee}$  defines model
- Time-dependent density functional theory (TDDFT) provides foundation

## Few-electron problem II

Choices:

$$\begin{aligned} v_{ee}(\mathbf{r}, t) &= v_{ee}^0(r) && \text{no response} \\ v_{ee}(\mathbf{r}, t) &= f(t)v_{ee}^0(r) && \text{global target response*} \\ (v_{ee}(\mathbf{r}, t) &= v_{ee}[\psi_j](\mathbf{r}, t)) && \text{microscopic response} \end{aligned}$$

- TC-BGM can be used within IEM
- Model uncertainties (IEM)  $\leftrightarrow$  numerical uncertainties (convergence etc.)

\* Kirchner *et al.*, PRA 2000

## Few-electron problem III

Single-particle solutions → many-electron info

- Option 1: IEM (multinomial) analysis  
e.g. single and double capture for  $N = 2$ :

$$P_1 = 2p_{\text{cap}}(1 - p_{\text{cap}})$$

$$P_2 = p_{\text{cap}}^2$$

- Option 2: determinants (density matrices)\*
- Further options: correlation integrals\*\*, single-active electron model, ...

analysis = source of model uncertainties

\*Lüdde and Dreizler, JPB 1985; \*\*Baxter and TK, PRA 2016

# Two-Center Basis Generator Method

Expansion (of single-particle solutions) in  
**dynamically adapted** model space

- Start with bound target and projectile states

$$\phi_v^0(\mathbf{r}) = \begin{cases} \phi_v(\mathbf{r}_t) \exp(i\mathbf{v}_t \cdot \mathbf{r}) & \text{if } v \leq V_t \\ \phi_v(\mathbf{r}_p) \exp(i\mathbf{v}_p \cdot \mathbf{r}) & \text{else} \end{cases}$$

- Add pseudo states

$$\chi_v^\mu(\mathbf{r}, t) = [W_p(t)]^\mu \phi_v^0(\mathbf{r}) \quad v = 1, \dots, V_t$$

- Expand

$$|\psi_i(t)\rangle = \sum_{v,\mu} c_{v,\mu}^i(t) |\chi_v^\mu(t)\rangle$$

- TC-BGM = TCAO + (special) pseudo states

**3. Study #1: collisions with  
hydrogen vs. krypton atoms**

# Objective: Lyman-line emissions

- Radiative transitions of one electron in excited projectile state
- Consider pure single capture and autoionizing double capture<sup>1</sup>
- Solve rate equations

$$\frac{dN_p(t)}{dt} = \sum_{i=p+1}^m N_i(t) A_{i \rightarrow p} - N_p(t) \sum_{f=1}^{p-1} A_{p \rightarrow f}$$

- Total photon counts

$$(\text{counts})_{nl \rightarrow n'l'} = A_{nl \rightarrow n'l'}^{\text{rad}} \int_0^{\infty} N_{nl}(t) dt$$

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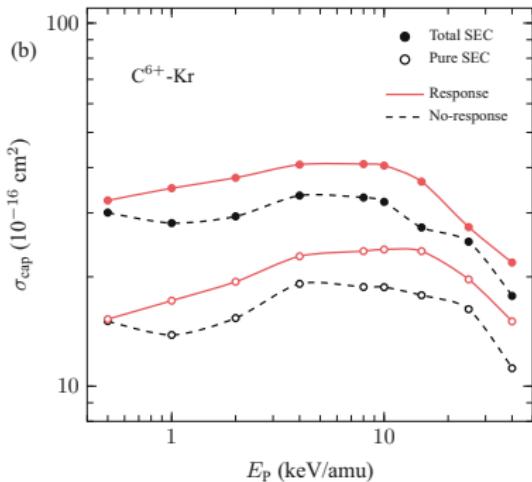
<sup>1</sup>Auger rates calculated with RATIP by S. Fritzsche (2001)

# Some basis parameters

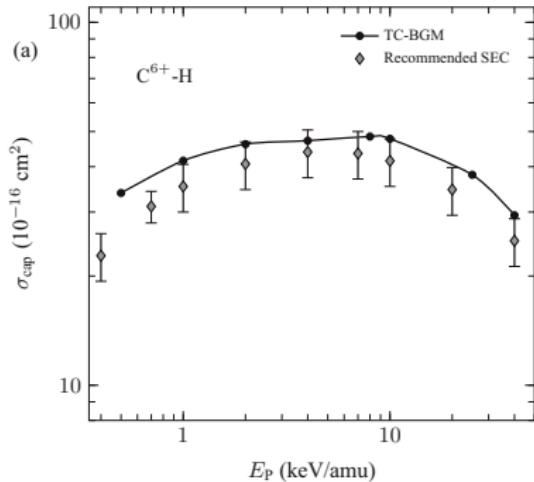
- Projectile states:  $n = 1, \dots, 7$  ( $\sum = 84$ )
- Target states
  - hydrogen:  $KLMN$  shell states ( $\sum = 20$ )
  - krypton:  $MNO$  shell states ( $\sum = 31$ )
- BGM pseudo states
  - hydrogen: up to 52
  - krypton: up to 66

# $\text{C}^{6+}$ -impact: total capture

krypton



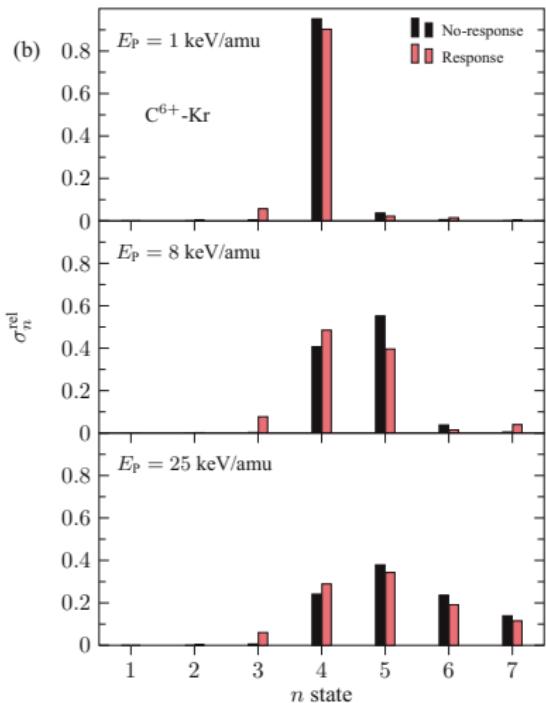
hydrogen



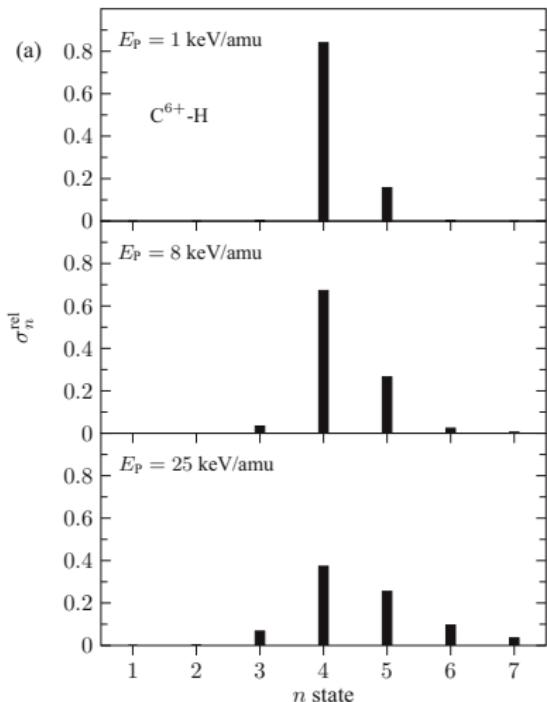
Recommended values: Janev *et al.*, ADNDT 1988  
Leung and Kirchner, PRA **97**, 062705 (2018)

# $\text{C}^{6+}$ -impact: n-state distributions

krypton



hydrogen



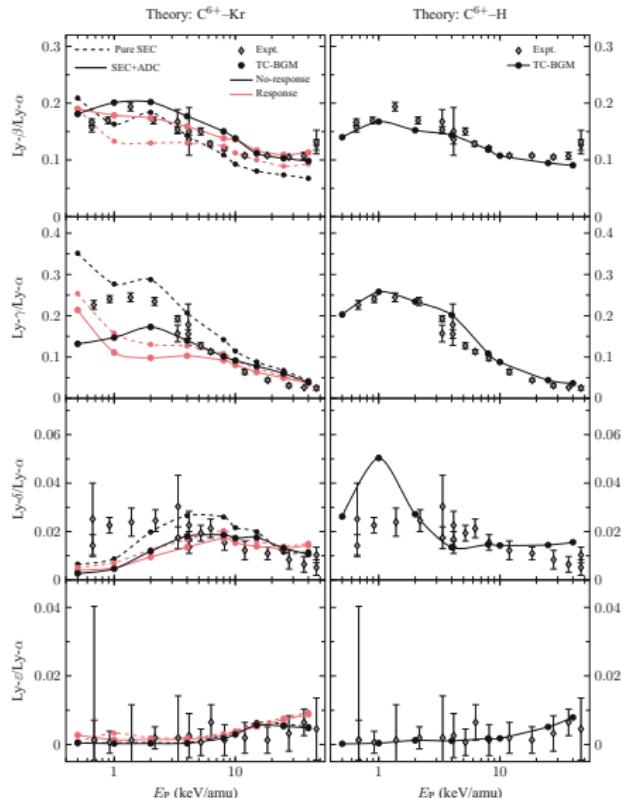
# $\text{C}^{6+}$ -impact: Lyman-emission ratios

$$\frac{3p \rightarrow 1s}{2p \rightarrow 1s}$$

$$\frac{4p \rightarrow 1s}{2p \rightarrow 1s}$$

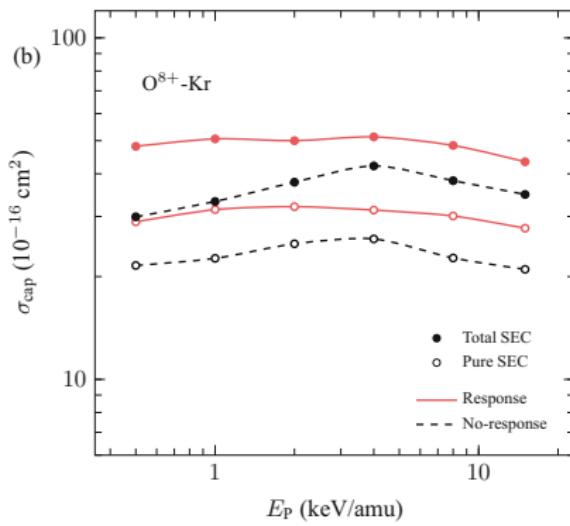
$$\frac{5p \rightarrow 1s}{2p \rightarrow 1s}$$

$$\frac{6p \rightarrow 1s}{2p \rightarrow 1s}$$

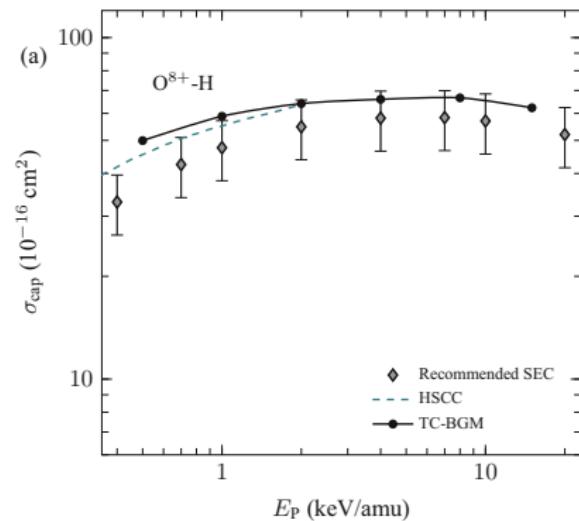


# $O^{8+}$ -impact: total capture

krypton



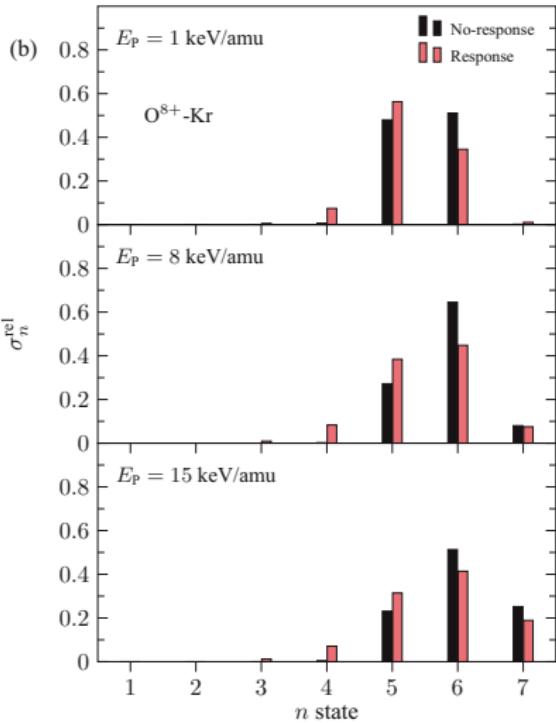
hydrogen



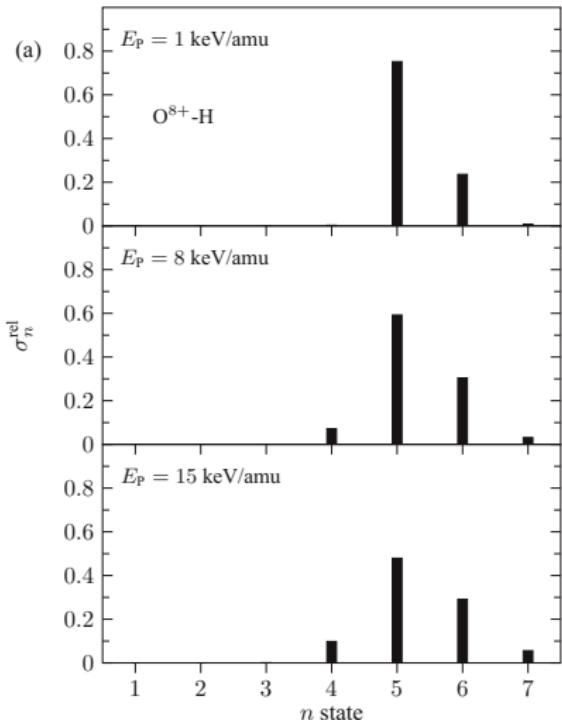
Rec.: Janev *et al.*, ADNDT 1988; HSCC: Lee *et al.*, PRA 2004  
Leung and Kirchner, PRA **97**, 062705 (2018)

# $O^{8+}$ -impact: n-state distributions

krypton



hydrogen



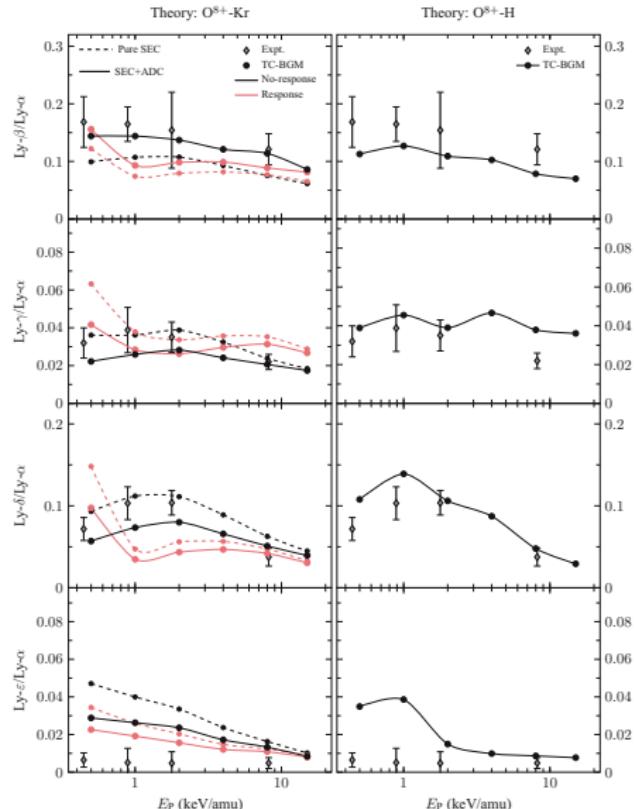
# $O^{8+}$ -impact: Lyman-emission ratios

$$\frac{3p \rightarrow 1s}{2p \rightarrow 1s}$$

$$\frac{4p \rightarrow 1s}{2p \rightarrow 1s}$$

$$\frac{5p \rightarrow 1s}{2p \rightarrow 1s}$$

$$\frac{6p \rightarrow 1s}{2p \rightarrow 1s}$$



**4. Study #2: collisions with excited hydrogen atoms**

# Some basis parameters

- Projectile states:  $n = 1, \dots, 6$  ( $\sum = 56$ )
- Target states
  - n=5 basis:  $KLMNO$  shell states ( $\sum = 35$ )
  - n=6 basis:  $KLMNOP$  shell states ( $\sum = 56$ )
- BGM pseudo states
  - n=5 basis: up to 94
  - n=6 basis: up to 106

Compare results for p-H(2s) with:

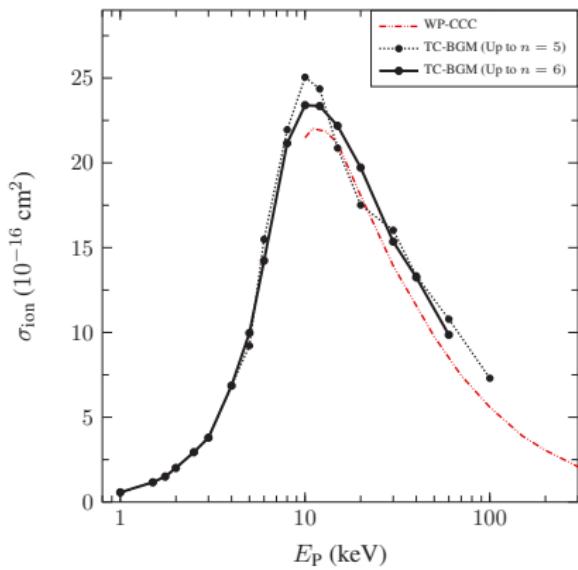
Abdurakhmanov *et al.*, Plasma Phys. Control. Fusion **60**, 095009 (2018)

Pindzola *et al.*, Phys. Rev. A **72**, 062703 (2005)

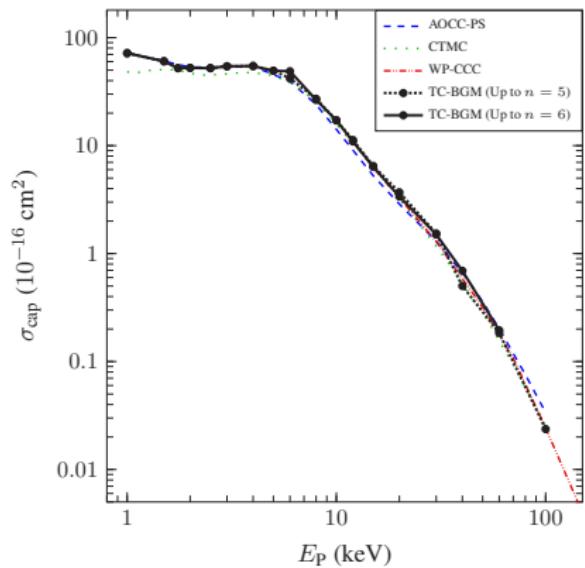
(see also Taouitioui *et al.*, J. Phys. B **51**, 235202 (2018))

# p-H(2s): total ionization and capture

## ionization



## capture



WP-CCC: Abdurakhmanov *et al.*, Plasma Phys. Control. Fusion 2018  
AOCC-PS and CTMC: Pindzola *et al.*, Phys. Rev. A 2005

# Summary and outlook

(TC)-BGM: coupled-channel method within semiclassical approximation

## Hydrogen vs krypton targets

- Krypton is not a good surrogate for hydrogen
- Need a better response model for more reliable/accurate Kr calculations

## Excited hydrogen targets

- Higher demands on basis sets and sizes
- Overall good agreement with WP-CCC
- Move on to H(2p)